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Thermal Radiative Effects on MHD Casson Nanofluid Boundary Layer Over a Moving Surface

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The present paper investigated the characteristics of flow and heat transfer of a Casson nanofluid boundary layer over a continuous moving surface subjected to thermal radiation and magnetic field. Nanosize particles (CuO, Ag, and Al_2O_3) are tested. Method of similarity transformation used to convert the governing equations of the boundary layer to a nonlinear ordinary differential equations system whose solved analytically at steady flow state and numerically at the unsteady flow boundary layer. The obtained results for the Nusselt number, skin friction, as well as for the temperature and velocity of the boundary layer for selected values of the controlling parameters. Results were compared with previously published research in some special cases and get it in a good agreement and the results were tabulated and graphs showed a good and significant effect of heat transfer by injection of the solid materials.

KEYWORDS: Casson Fluids, Nanofluids, Thermal Radiative, MHD.

1. INTRODUCTION

From a technological point of view, we have to pay much attention to studying the flow of a boundary layer around a moving surface. Importance of fluid flow around a moving surface can be sense for any inevitable increase in industrial applications, application of medical, and industry of biomedical, power generation for nuclear reactors and new technology.

The ongoing research will concentrate on the nanofluids that can be exploited in fuel cells, hybrid powered engines, vehicle thermal management, microelectronics, pharmaceutical processes, heat exchanger, engine cooling, nuclear reactor coolant, and boiler flue gas temperature reduction as noted by Sheikholeslami in his book.[1]

It is known that traditional fluids of heat transfer like water, oil, and ethylene glycol mixture are poor fluids in heat transfer; such fluids play important roles between heat transfer of the surface and heat transfer of the medium in the heat transfer process. Advanced technique, which uses a mixture of the base fluid and nanoparticles, the resulting of the mixing between the base fluid and nanoparticles having unparalleled chemical and physical properties is called a nanofluid. Its magnitude of physical properties studied and tabled in the Simulation Numerically of Water-Alumina Nanofluid in Geometry of Subchannel by Nazififard.[2] The presence of nanoparticles in nanofluid is expected to increases thermal conductivity, thus greatly and enhancing the heat transfer properties of nanofluid indicated by Olanrewaju et al.[3] Studied nanofluids flow boundary layer around a moving surface by Bachok et al.[4] Study was conducted on the flow of boundary layer of a nanofluid over a stretching sheet with a boundary condition convective by Gbadeyan.[5]

The radiation effect becomes necessary, If the difference is large between the ambient temperature and the surface, Investigated a nanofluid viscous flow and heat transfer characteristics Past a stretching nonlinearly sheet in the attendance of thermal radiation by Hady et al.[6] Olanrewaju et al.[7] investigated the boundary layer of nanofluids flow around a moving surface in the attendance of radiation. Studied the flow and heat transfer of a Variable thickness of a moving surface has a nonlinear velocity in the presence of radiaton by Elboshbeshy.[8]

Many geophysical and engineering applications depends on MHD boundary layers flow of mass and heat transfer past a flat surfaces such as enhanced cooling of nuclear reactors, recovery of oil and the thermal insulation explained effects of Magnetic field of nanofluid flow with free convection around a semi-infinite vertical flat plate by Hamad et al.[9] Furthermore, Applications of the magnetic field of circulatory system in the human artery and its application to the treatment of some disorders of cardiovascular were studied by Sharma.[10] Hamad et al.[11] studied the effects of magnetic field on nanofluid which has a free convection flow over a semi-infinite vertical plate.

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Khan[12] investigated the boundary layer of a nanafluid has an unsteady free convection flow past a stretching sheet with a magnetic field in the presence of thermal radiation. Recently, Shanker et al.[13] studied MHD flow with melting heat transfer of nanofluids past a permeable stretching sheet. Elbashbeshy et al.[14] obtain an exact solution of boundary layer nanofluid flow past a moving surface in the attendance of magnetic field.

Another kind of non-Newtonian fluid is called as a Casson fluid. Yield stress is exhibits in Casson fluid. Casson fluid is known as a shear thinning fluid which is assumed to be infinity viscosity at a shear rate equal zero, and the yield stress without any flow occurs, at the shear rate of infinity the value of viscosity becomes zero. If a yield stress is more than shear stress is applied to the fluid, it acts like a solid. While if the yield stress is less than shear stress applied, it begins to move. Casson fluid Examples are of the type as follows: honey, jelly, soup, concentrated fruit juices, tomato sauce, etc. Human blood can also be considered as a Casson fluid. Boundary layer of MHD Casson fluid flow around a stretched sheet Studied by Hayat et al.[15] Ananalytical solution of MHD Casson fluid around a permeable stretching sheet reported by Bhattacharyya et al.[16] A similarity solution of a flow of MHD Casson fluid past a accessible shrinking sheet studied by Thiagarajan.[17] Recently, a flow of an unsteady Casson fluid over a stretching surface with thermal radiation and cross diffusion studied chemical reacting Numerically by Pushpalatha.[18] Imran et al.[19] investigated the casson fluid MHD convective flows have a slip and a radiative heat transfer on the boundary.

All of these studies compact with studying a nanofluid flow and heat transfer over a continuous constant or variable thickness surface subjected to thermal radiation and nonlinear velocity, Or studying the boundary layer of a Casson fluid MHD flow affected by thermal radiation and heat transfer over a stretching sheet. Therefore the aim of this paper is to study Casson nanofluid over a continuous surface subjected to magnetic field and thermal radiation, As a cooling process simulation.

2. PROBLEM FORMULATION

Assume a two dimensional unsteady flow of an incompressible casson nanofluid around a continuous and moving surface. Assumed that the main fluid is water contains one of these nanoparticles, aluminum oxide (Al_2O_3), Copper oxide (CuO) or silver (Ag), we assumed that the nanoparticles have a uniform size and form. We assumed that a surface moving with nonlinear velocity $U_w(x, t) = aX/(1-ct)$ within an unsteady and incompressible flow of casson nanofluid, cooling medium consists of nanoparticles and water base fluid. The fluid subjected to nonlinear transverse magnetic field with strength of B_0 and nonlinear thermal radiation with heat flux q_r. Moreover, as shown in Figure 1, the surface temperature assumed T_w and ambient temperature of the boundary layer taken T_∞.

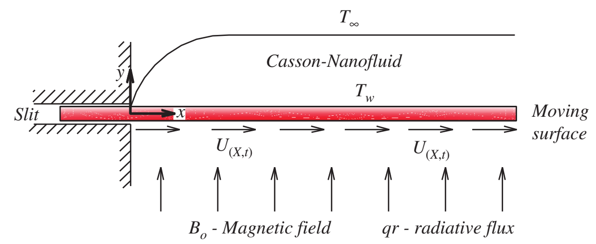

Fig. 1. Physical model and coordinate system.

The equations of motion (continuity equation, momentum equation and energy equation) of the governing boundary layer describing the unsteady casson nanofluid two-dimensional hydromagnetic flow around a moving surface and subjected nonlinear Rosseland thermal radiation are written as:-[14]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{1}{\beta}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho(C_P)_{nf}}\frac{\partial q_r}{\partial y} \quad (3)$$

Where u is the component of velocity in the x direction and v is the component of velocity in the y direction, The fluid is considered to absorb radiation. We define the relative heat flux in the energy equation by using Rosselond approximation

$$q_r = -\frac{4\sigma^* \partial T^4}{3\alpha^* \partial y} \quad (4)$$

Where α^* and σ^* are the mean absorption coefficient and the Stefen-Boltzman constant respectively, we can define T^4 as a linear function of temperature, expending T^4 in the Taylor series about T_∞ We get

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \quad (5)$$

Using (4) and (5) in energy Eq. (3), We get

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\alpha(\rho C_P)_{nf}}\frac{\partial^2 T}{\partial y^2} \quad (6)$$

Using boundary conditions

$$\begin{array}{ll} v = 0,\ u = U_w,\ T = T_w & \text{at } y = 0 \\ v = 0,\ u = 0,\ T = T_\infty & \text{at } y \to \infty \end{array} \quad (7)$$

The velocity and temperature surface are assumed in the form

$$U_w(x, t) = \frac{ax}{(1-ct)} T_w(x, t) = T_\infty + \frac{bx}{(1-ct)} \quad (8)$$

where a, b and c are constants.

Table I. Physical properties of water and the nanoparticles Ag, Cuo, and Al_2O_3.

Properties	Fluid (water)	Cuo	Ag	Al_2O_3
C_p (J/Kgk)	4179	551	235	765
K (W/mK)	0.613	32.9	429	40
$\alpha \times 10^7$ (m^2/s)	1.47	50.2	1738.6	131.7
ρ (Kg/m^3)	97.1	6320	10500	3970

Nanofluid properties are defined from Ishak[20] as follows:

$$\left\{\begin{array}{c} \mu_{nf} = \dfrac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \\ \alpha_{nf} = \dfrac{K_{nf}}{(\rho C_p)_{nf}} \\ (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\ \dfrac{K_{nf}}{K_f} = \dfrac{(K_s+2K_f)-2\phi(K_f-K_s)}{(K_s+2K_f)+\phi(K_f-K_s)} \end{array}\right\}$$

Where μ_{nf} is the nanofluid dynamic viscosity, ρ_{nf} is the nanofluid density, α_{nf} defined as the diffusion of the nanofluid, C_p is the nanofluid specific heat, T is the nanofluid temperature, T_w is the surface temperature, T_∞ is ambient temperature and ϕ is nanoparticle volume friction.

3. SIMILARITY TRANSFORMATION

In this section, we will apply a similarity solution to transform the Eqs. (1), (2), and (6) whose subjected to the boundary conditions (7) to an ordinary differential equations system using the following dimensionless functions:

$$\varphi = \sqrt{\frac{x^2}{1-ct} a v_f} F(\eta) \quad \eta = y\sqrt{\frac{a}{v_f(1-ct)}} \tag{9}$$
$$T = T_\infty + (T_w - T_\infty)\theta(\eta)$$

Where φ is the stream function, θ is the dimensionless temperature η is the similarity variable and v_f is the kinematic viscosity of the water, we can define the stream function φ as:

$$u = \frac{\partial\varphi}{\partial y}, \quad v = -\frac{\partial\varphi}{\partial x}$$

Substituting (8) into (2) and (6) we get the following ordinary differential equations

$$\left(\frac{1}{(1-\emptyset)^{2.5}D} + \frac{1}{\beta}\right) f''' + ff'' - f'^2 - Mf' - A\left(f' + \frac{1}{2}\eta f''\right) = 0 \tag{10}$$

$$\left(\frac{4R+3\gamma l}{3\gamma P_r}\right)\theta'' + f\theta' - f'\theta - A\left(\theta + \frac{1}{2}\eta\theta'\right) = 0 \tag{11}$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad f'(\infty) = 0, \quad \theta(\infty) = 0 \tag{12}$$

Where

$$A = \frac{c}{a}, \quad M = \frac{\sigma B_0^2}{\rho a}, \quad \beta = \frac{\mu_B\sqrt{2\pi_c}}{P_y},$$

$$D = \left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right), \quad \gamma = \left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right),$$

$$l = \frac{(K_{nf}/K_f)}{[1-\phi+\phi(\rho C_p)_s/\rho C_p)_f)]}, \quad R = \frac{4\sigma T_\infty^3}{\alpha K_f}, \quad P_r = \frac{(\mu C_p)_f}{K_f}$$

Where A is the Parameter of the unsteadiness, β is the casson fluid parameter, R is the radiation parameter, P_r is the Prandtl number P_y is the fluid yield stress, π_c is the critical value of this product based on a non-Newtonian model, μ_B is the viscosity of the plastic dynamic of the non-Newtonian fluid and μ is the fluid dynamic viscosity.

4. SOLUTIONS AND RESULTS

4.1. State 1: Boundary Layer of a Steady Flow

For boundary layer of a steady flow, we consider unsteadiness Parameter $(A) = 0$. Thus, Eqs. (10)–(11) become

$$\left(\frac{1}{(1-\emptyset)^{2.5}D} + \frac{1}{\beta}\right) f''' + ff'' - f'^2 - Mf' = 0 \tag{13}$$

$$\left(\frac{4R+3\gamma l}{3\gamma P_r}\right)\theta'' + f\theta' - f'\theta = 0 \tag{14}$$

4.2. Momentum Equation Exact Solution

The Eq. (13) solution assumed to be in the form

$$f(\eta) = \frac{(1-e^{-z\eta})}{z} \tag{15}$$

This is satisfied with the following boundary conditions:

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0 \tag{16}$$

Put (15) in (13) using boundary conditions (16), one can obtain

$$z = \frac{\sqrt{4(K+M)}}{2K} \tag{17}$$

Where

$$K = \frac{\beta + (1-\phi)^{2.5}D}{\beta D(1-\phi)^{2.5}} \tag{18}$$

4.2.1. Thermal Boundary Layer Problem Exact Solution

Exchanging in Eq. (14) by Eq. (15), one can getting the following ordinary differential equation:

$$\theta'' + \left(\frac{3\gamma P_r}{4R+3\gamma l}\right)\left[\frac{1}{z}(1-e^{-z\eta})\theta' - (e^{-z\eta})\theta\right] = 0 \tag{19}$$

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With boundary conditions given by

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{20}$$

Equation (19) is the second order homogenous linear ordinary differential with variable coefficients that can be solved by entering the change variable:

$$\vartheta = -S\frac{e^{-z\eta}}{z^2} \tag{21}$$

Where $S = (3\gamma P_r/(4R+3\gamma l))$, Consequently Eq. (19) can be written as

$$\vartheta\frac{d^2\theta}{d\vartheta^2} + (1-H-\vartheta)\frac{d\theta}{d\vartheta} + \theta = 0 \tag{22}$$

Where $H = S/z^2$

With boundary conditions

$$\left(\frac{S}{z^2}\right) = 1, \quad \theta(\infty) = 0 \tag{23}$$

Equation (22) takes kummer's differential equation shape, which has a function $1F1$ of kummer confluent hypergeometric as a solution:

$$\theta(\vartheta) = \left(\frac{\vartheta z^2}{S}\right)^H \frac{1F1(H-1; H+1; -\vartheta)}{1F1(H-1; H+1; -(S/z^2))} \tag{24}$$

Equation (22) solution in terms of η found as

$$\theta(\eta) = (e^{-z\eta})^H \frac{1F1(H-1; H+1; -\vartheta)}{1F1(H-1; H+1; -(S/z^2))} \tag{25}$$

As well, the gradient temperature of the surface is

$$\theta'(0) = -(zH) + \left(\frac{S(H-1)}{z(H+1)}\right) \times \frac{1F1(H; H+2; -(S/z^2))}{1F1(H-1; H+1; -(S/z^2))} \tag{26}$$

4.3. Case 2: Boundary Layer of an Unsteady Flow

Equations (10)–(11) and the boundary conditions (12) are solved numerically. In order to solve this system, we first need to find the missing initial conditions $\theta'(0)$ and $f''(0)$, by constructing a program for the system on initial value problem with missing initial conditions. To solve this system, We have to suggest the proper value of $\eta \to \infty'' \eta''_\infty$, and then, the missing values are calculated by the code using find root technique. Once the values of $\theta'(0)$ and $f''(0)$, have no change with an increase of $\eta \to \infty$ (i.e., boundary conditions $f'(\eta_\infty) = 0$, $\theta'(\eta_\infty) = 0$ satisfied at the suggested value of $(\eta \to \infty)$, becomes ready to solve the system using classical Runge–Kutta method from the fourth order with step size $\Delta\eta = 0.01$. To support the unsteady case results gained by this method, (Table II) used to compare the results for $\theta'(0)$ with the exact solution and the numerical solution obtained in this study and numerical solution of the study of unsteady MHD flow and heat transfer around a Stretching Plate which reported by Ishak et al.[21]

Table II. Values of $-\theta'(0)$ at $Rd = \phi = 0$, $\beta = \infty$ and various value of M and Pr.

			Present results	
M	Pr	Ishak[21]	Exact	Numerical
0	0.01	0.01970	0.01970	0.01972
0	0.72	0.80863	0.80863	0.80863
0	3.00	1.92368	1.92367	1.92368
0	6.70	3.00027	3.00025	3.00027
1	0.01	0.01403	0.01403	0.01403
1	0.70	0.68969	0.68969	0.68970
1	1.00	0.89214	0.89214	0.89214
1	10.0	3.61699	3.61698	3.61699

5. RESULTS

Nusselt number and the skin friction coefficient are the most important characteristic of flows are, which physically matched to rate of heat transfer and the surface shear stress. These characteristics have a direct effect on the properties of mechanical of the surface during the process of cooling, such as increasing the heat transfer rate from the surface accelerates the surface cooling, which improve shear strength of the surface and the hardness, but decrease its ductility and increases surface cracking.

5.1. Surface Shear Stress

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2\mu_{nf}}{\rho_f} = \frac{f''(0)}{X\sqrt{aV_f/(1-ct)}} \tag{27}$$

- The coefficient of skin friction is given by $C_{fx} = 2\tau_w/(\rho U_w^2)$.

Then, the skin friction

$$C_{fx} = \frac{2f''(0)}{\sqrt{R_e}(1-\phi)^{2.5}} \tag{28}$$

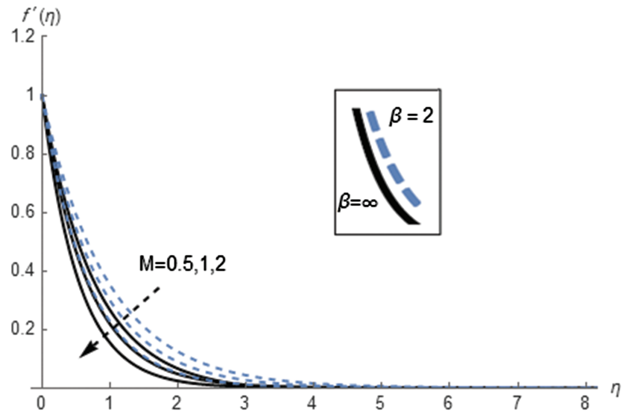

Fig. 2. Velocity profiles of the different values of magnetic parameter (M) $Pr = 6.2$, $R = 0.5$, $A = 0.2$, 10% (CuO water).

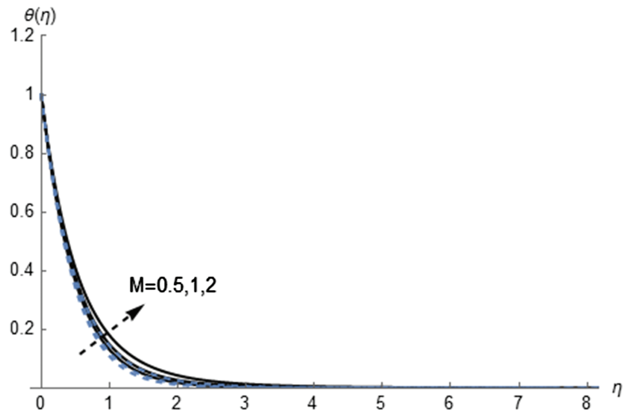

Fig. 3. Temperature profiles for different values of magnetic parameter (M) $Pr = 6.2$, $R = 0.5$, $A = 0.2$, 10% (CuO water).

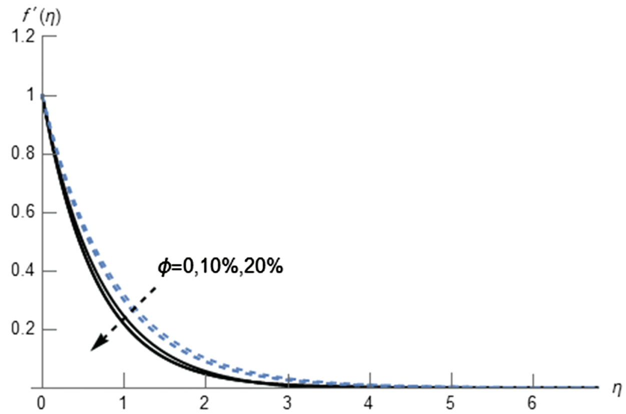

Fig. 4. Velocity profiles for different values of nanoparticle volume fraction (ϕ) $Pr = 6.2$, $R = 0.5$, $M = 1$, $A = 0.2$.

5.2. Surface Heat Flux

$$q_w = -K_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= -XK_{nf}\sqrt{\frac{a^3}{V_f(1-ct)^3}}\theta'(0) \tag{29}$$

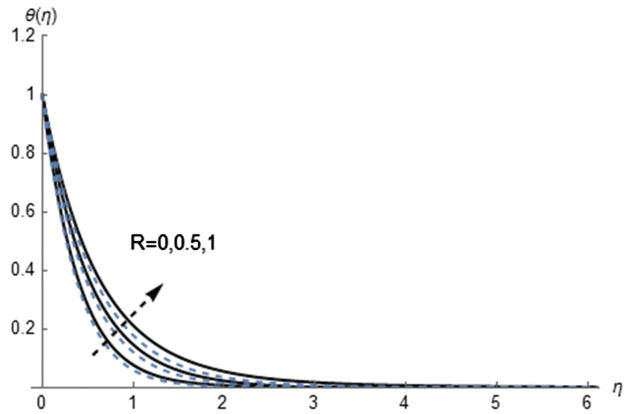

Fig. 6. Temperature profiles for different values of parameter of thermal radiation (R) $Pr = 6.2$, $M = 1$, $A = 0.2$, 10% (CuO-water).

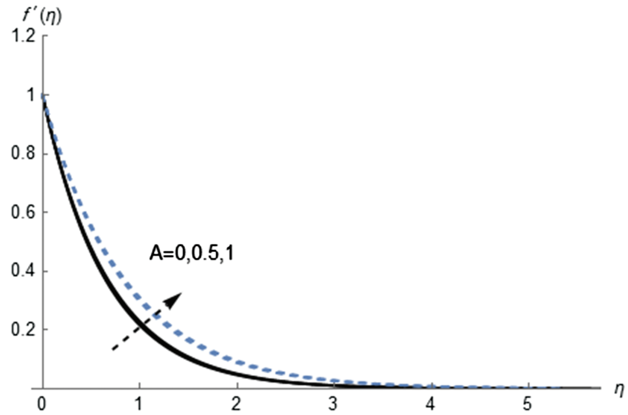

Fig. 7. Velocity profiles for different values of parameter of the unsteady (A) $Pr = 6.2$, $R = 0.5$, $M = 1$ 10% (CuO water).

- The Nusselt number (Nu) is given by $Xq_w/(K_f(T_w - T_\infty))$.

Then, Nusselt number

$$Nu = -\frac{K_{nf}}{K_f}\sqrt{R_e}\theta'(0) \tag{30}$$

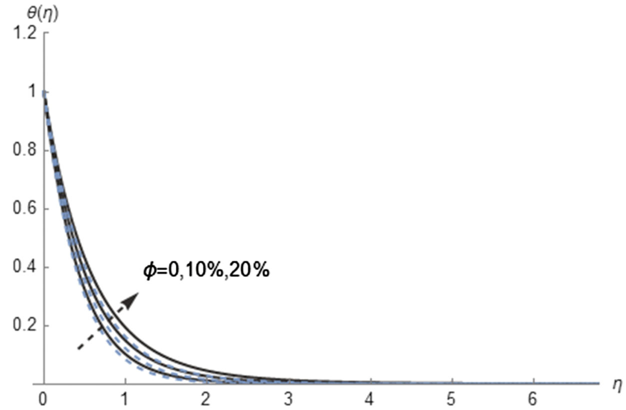

Fig. 5. Temperature profiles for different values of nanoparticle volume fraction (ϕ) $Pr = 6.2$, $R = 0.5$, $M = 1$, $A = 0.2$.

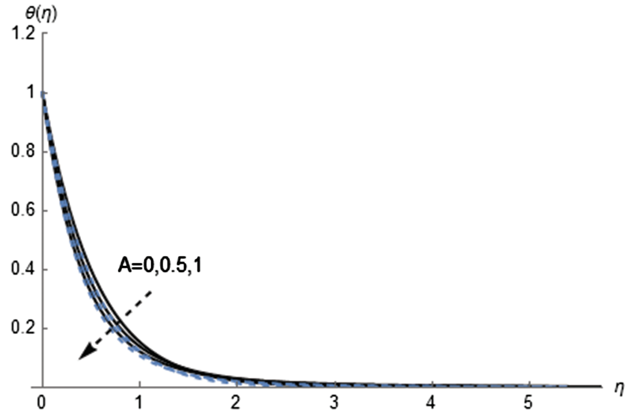

Fig. 8. Temperature profiles for different values of parameter of the unsteady (A) $Pr = 6.2$, $R = 0.5$, $M = 1$, 10% (CuO water).

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Table III. Values of temperature gradient and velocity gradient at the surface at $Pr = 6.2$, $R = 0.5$, $M = 1$, and $A = 0.2$ with various value of β and ϕ.

		CuO-water				Ag-water				Al_2O_3-water			
β	ϕ	$-f''(0)$	$-\theta'(0)$	$C_{fx}\sqrt{Re}$	$Nu/\sqrt{Re}$	$-f''(0)$	$-\theta'(0)$	$C_{fx}\sqrt{Re}$	$Nu/\sqrt{Re}$	$-f''(0)$	$-\theta'(0)$	$C_{fx}\sqrt{Re}$	$Nu/\sqrt{Re}$
∞	0	1.4063	2.1971	2.8126	2.1971	1.4063	2.1971	2.8126	2.1971	1.4063	2.1971	2.8126	2.1971
∞	0.1	1.5268	1.9422	3.9737	2.5511	1.7228	1.8590	4.4840	2.4758	1.4045	1.9557	3.6557	2.5755
∞	0.2	1.5299	1.7287	5.3455	2.9390	1.8138	1.5945	6.3373	2.7841	1.3443	1.7441	4.6968	2.9800
2	0	1.1482	2.2567	2.2964	2.2567	1.1482	2.2567	2.2964	2.2567	1.1482	2.2567	2.2964	2.2567
2	0.1	1.2111	2.0148	3.1521	2.6465	1.3021	1.9548	3.3892	2.6034	1.1473	2.0151	2.9861	2.6538
2	0.2	1.2127	1.7811	4.2369	3.0622	1.3401	1.6998	4.6824	2.9673	1.1137	1.7971	3.8913	3.0707

6. DISCUSSION

This paper presented a model of a mathematical of a continuous moving surface of MHD casson nanofluid in the attendance of thermal radiation. Temperature and velocity of the CuO nanofluid boundary layer affected by Magnetic parameter (M) is shown in Figures 3 and 4. Obviously increasing M increases the fluid temperature and reduces the velocity.

The effect of nanoparticle volume fraction (ϕ) on the temperature and velocity of the CuO nanofluid boundary layer is shown in Figures 4 and 5. It is found that increasing ϕ increases the fluid temperature and decreases the velocity.

The effect of thermal radiation parameter (R) on the temperature of the CuO nanofluid boundary layer is shown in Figure 6. It is show that increasing R decreases the velocity and increases the fluid temperature. The effect of unsteady parameter (A) on the temperature and velocity of the CuO nanofluid boundary layer is shown in Figures 7 and 8. It is show that increasing A increases the velocity and decreases the fluid temperature.

Casson fluid parameter (β) effect on the temperature and velocity of the CuO boundary layer nanofluid is through in Figures 2–8. It is show when $\beta = \infty$ increases the fluid temperature and decreases the velocity, and when β equal a known magnitude as 2 increases the velocity and decreases the fluid temperature.

Table III view values of the temperature and velocity at the surface and the corresponding values of Nusselt number and skin friction of different values of ϕ and β. It is clear that the velocity at the surface increases gradually by changing the nanoparticle at all casson fluid conditions from Al_2O_3 to CuO to Ag. But the temperature of the surface increased gradually by changing the nanoparticle from Ag to CuO to Al_2O_3 at all casson fluid conditions.

It also show that the surface shear stress and skin friction are lower in the situation of Al_2O_3 than that in the situation of Ag and CuO nanofluid at all casson fluid conditions, also the heat transfer rate and Nusselt number in the case of Al_2O_3 are higher than that in the case of CuO and Ag in casson nanofluid at all casson fluid conditions, Which means that using Al_2O_3 is useful to improve the strength and hardness more than the other nanoparticles and we can use it as a cooling medium.

Table III show also that the surface velocity values increases gradually when the particle volume friction increase in case of Cuo and Ag and decrease for Al_2O_3. And the temperature increase when the particle volume fraction increase in all nanoparticles at all casson fluid conditions. So that the nanofluid with 20% nanoparticles is better than 10% nanoparticles to improving the mechanical properties, and when we increase this percent more and more it leads to accelerated surface cooling and improves the hardness and the strength of the surface.

Furthermore Table III confirm that when we use casson nanofluid the value of gradient velocity at the surface decreases. But the temperature of the surface increases. It also show that the surface shear stress and skin friction decreases when we used casson nanofluid But the rate of heat transfer and Nusselt number increases. So that using Casson nanofluid gives a good results and improves the surface strength and hardness and accelerated cooling of the surface.

7. CONCLUSION

- Using Casson nanofluid gives good results and improves the surface strength and hardness and accelerated cooling of the surface.
- Increasing the percentage of nanoparticles, improve the mechanical properties, By improving the surface strength and hardness and accelerating the cooling of the surface.
- The best type used to increase the heat transfer and to decrease the surface shear stress is Al_2O_3.

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